[Rewais, 8(4): April 2021] ISSN 2348 – 8034
DOI: https://doi.org/10.29121/gjesr.v8.i04.2021.1 Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

# USING POTENTIAL DEPENDENT KLEIN GORDON EQUATION TO FIND MAGIC QUANTUM NUMBER

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## ABSTRACT

Potential dependent energy- momentum relativistic relation has been used to derive new quantum Klein- Gordon equation. This equation reduces to ordinary Klein- Gordon equation in the absence of potential. Treating nucleons as strings anew energy-quantized expression has been found. This energy resembles that of Schrödinger harmonic oscillator with additional term representing the rest mass. This model predicts the magic numbers.

***Keywords:*** *Potential, momentum, Klein Gordon, string, magic numbers.*

## I. INTRODUCTION

Atoms are the building blocks of matter. The block matter can be described by using classical laws like Newton laws and Maxwell's equations [1].the experiments done that they consist of small tiny particles revolving around the nucleus known as electrons. The nucleus consists of protons and neutrons of almost equal numbers. The excitation of these atoms by any energy source causes them to emit electromagnetic radiation. One of the important atomic radiations is so called black body radiation, due to excitation by heat. The spectra of black body, is the first challenge that shows the failure of classical laws in describing the behavior of the atomic world [2]. This encourages Max Plank to propose a new concept to describe the black body radiation. He proposes that light is emitted as discrete quanta called photons. The energy of each quanta is directly proportional to the light frequency. This new quantum concept opens a new horizon in physics. It encourages De Brogglie to prose also that particles like electrons behave as discrete quanta. This new concept of quanta encourages Schrodinger and Heisenberg in dependently, to formulate the so called quantum laws [3, 4]. These quantum laws open a new era in physic s and succeed in explaining a large number of atomic phenomena.

Later on Schrodinger equation, which is based on classical Newton energy for slow particles, has been promoted to describe fast particle. Klein –Gordon and Dirac made this development where they formulated the so-called relativistic quantum mechanics [5]. Klein nonlinear equation describes spin less bosons, while Dirac linear one describes fermions [6, 7].

Although both relativistic equation describe fast particle, but they suffer from the lack of a simple expression which recognized all potentials. Another problem is relate to the fact that, ion no single equation can describe the behavior of bosons at the same time. Different attempts have made to modify quantum relativistic equation to widen their scope in describing physical phenomena [8].The relativistic modified version of Nagua [9]. Utilized the quantum relativistic equation beside periodicity condition to find the harmonic oscillator solution. A paper published by Fatima [10].

Showed that relativistic potential dependent equation derived from Generalized Special Relativity (GSR) can described the behavior of bio photons as well as photons propogated in free space.

All this success of GSR motivates in trying to describe both bosons and fermions using only Klein Gordon equation by adopting certain approximations and approaches. This had done in section (2). Section (3) has devoted for conclusion.

## II. MODEL AND DISCUSSIONS

Consider a particle moving with repulsive electric field a way from afield source. This resembles the case of motion of an electron away from a negatively charged capacitor , where its field is uniform .In this case the final speed v can be written in terms of the initial speed $v_0$ and the acceleration a, beside displacement x in the from

$$v^2 = v_o^2 + 2ax \quad (1)$$

However, the force per unit mass is defined

$$F = -\frac{\partial\phi}{\partial x} = a \quad (2)$$

Thus:

$$\phi = \int d\phi = -\int adx$$

For uniform acceleration

$$\phi = -\text{ax} \quad (3)$$

Thus equation ( ) gives

$$v^2 = v_o^2 + 2ax = v_o^2 - 2\phi$$

$$v_o^2 = v^2 + 2\phi \quad (4)$$

Using the ordinary SR mass formula and inserting equation (4) gives

$$E = mc^2 = \frac{m_oc^2}{\sqrt{1-\frac{v_o^2}{c^2}}} = \frac{m_oc^2}{\sqrt{1-\frac{(v^2+2\phi)}{c^2}}} \quad (5)$$

$$E = \frac{m_oc^2}{\sqrt{\frac{m^2c^4-m^2v^2c^2-2m^2\phi c^2}{m^2c^4}}}$$

$$E = \frac{m_oc^2E}{\sqrt{E^2 - P^2C^2 - 2VE}}$$

$$E^2 - P^2C^2 - 2EV = m_o^2c^4$$

$$E^2 = P^2C^2 + 2EV + m_o^2c^4 \quad (6)$$

Where the potential V is related to the potential per unit mass $\phi$ according to the relation.

$$V = m\phi \quad (7)$$

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)} \quad (8)$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = E^2\psi$$

$$\frac{\hbar}{i}\nabla\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial x} = p\psi$$

$$-\hbar^2\nabla^2\psi = p^2\psi \quad (9)$$

Multiplying both sides of (6) by $\psi$ gives:

$$E^2\psi = P^2C^2\psi + 2VE\psi + m_o^2c^4\psi \quad (10)$$

A direct substitution of equation (9) gives

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -C^2\hbar^2\nabla^2\psi + 2i\hbar V\frac{\partial\psi}{\partial t} + m_o^2c^4\psi \quad (11)$$

To simplify the equation consider the solution:

$$\psi(r,t) = \psi = e^{-i\omega t}\phi(r)$$

$$\frac{\partial\psi}{\partial t} = -i\omega_o\psi \qquad \frac{\partial^2\psi}{\partial t^2} = -\omega_o^2\psi$$

$$\nabla^2\psi = e^{-i\omega_o t}\nabla^2\phi \quad (12)$$

Inserting equation (12) in (11) gives:

$$\hbar^2\omega_o^2\, e^{-i\omega_o t}\psi = [-C^2\hbar^2\nabla^2\phi + 2\left(\hbar\omega V\frac{\partial\psi}{\partial t} + m_o^2c^4\right)\phi]e^{-i\omega_o t} \quad (13)$$

Let

$$\hbar\omega_o = E \quad (14)$$

To get:

$$-C^2\hbar^2\nabla^2\phi + 2EV\phi + m_o^2c^4\phi = E^2\phi$$

$$\nabla^2\phi + \frac{E^2\phi}{C^2\hbar^2} - \frac{2EV\phi}{C^2\hbar^2} - \frac{m_o^2c^4\phi}{C^2\hbar^2} = 0 \quad (15)$$

For simplicity, let us consider

$$\frac{E^2 - m_o^2c^4}{C^2\hbar^2} = \frac{p^2c^2}{C^2\hbar^2} = \frac{p^2}{\hbar^2} = \frac{(\hbar K)^2}{\hbar^2} = K^2$$

$$C_o = \frac{2E}{C^2\hbar^2} \quad (16)$$

To get

$$\nabla^2\phi - C_oV\phi + K^2\phi = 0 \quad (17)$$

In spherical coordinate

$$\nabla^2\phi = [\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2 sin\theta^{-2}}\frac{\partial^2\phi}{\partial\phi^2}] \quad (18)$$

Inserting (18) in (17) and writing $\phi(r,\theta,\phi)$

In the form

$$\phi(r,\theta,\phi) = R(r)Y(\theta,\phi) \quad (19)$$

Yields

$$= \frac{1}{r^2}[\frac{y}{yR}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{R}{Rysin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial y}{\partial}\right) + \frac{R}{Rysin^2}\frac{\partial^2 y}{\partial\phi^2} - c_oV + K^2 = 0 \quad (20)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) - c_or^2V + k^2r^2 = -\frac{1}{ysin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial y}{\partial\theta}\right) - \frac{1}{ysin\theta^2}\frac{\partial^2 y}{\partial\phi^2} = C_2 \quad (21)$$

$$C_2 = \mathcal{L}(\mathcal{L}+1) \quad (22)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + k^2r^2 - c_or^2V = C_2 \quad (23)$$

For simplicity let

$$u = rR \qquad \frac{dR}{dr} = \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} \quad (24)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) = \frac{\partial}{\partial r}\left(r^2\left(\frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right)\right) = \frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} - u\right) = r\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} - \frac{\partial u}{\partial r} = \frac{r\partial^2 u}{\partial r^r} \quad (25)$$

$$r\frac{\partial^2 u}{\partial r^2} + (k^2r^2 - c_or^2V)\frac{u}{r} = C_2\frac{u}{r}$$

$$r\frac{\partial^2 u}{\partial r^2} + (k^2 - c_oV)ru = C_2\frac{u}{r} \quad (26)$$

$$\frac{\partial^2 u}{\partial r^2} + \left(k^2 - c_oV - \frac{C^2}{r^2}\right)u = 0 \quad (27)$$

$$u^{//} + \left(k^2 - c_oV - \frac{C^2}{r^2}\right)u = 0 \quad (28)$$

For a harmonic oscillator oscillating in a radial direction:

$$V = \frac{1}{2}k_or^2 \quad (29)$$

Equation (28) becomes

$$u^{//} + \left(k^2 - \frac{c_ok_o}{2}r^2 - \frac{C^2}{r^2}\right)u = 0 \quad (30)$$

Further simplification can be made by defining

$$y = \alpha r$$

$$u^{"} = \frac{d^2u}{dr^2} = \alpha^2\frac{d^2u}{dy^2} = \alpha^2\ddot{u} \quad (31)$$

$$\alpha^2\ddot{u} + \left(k^2 - \frac{c_ok_o}{2\alpha^2}y^2 - \frac{C^2}{y^2}\right)u = 0$$

$$\ddot{u} + \left(\frac{k^2}{\alpha^2} - \frac{c_ok_o}{2\alpha^4}y^2 - \frac{C^2}{y^2}\right)u = 0 \quad (32)$$

$\alpha$ Can be adjusted to be such that

$$\frac{c_ok_o}{2\alpha^4} = 1 \quad (33)$$

In view of equation (16) and the definition of $k_o$

$$\alpha^4 = \frac{c_o k_o}{2} = \frac{E}{C^2\hbar^2} k_o = \frac{mc^2(m\omega^2)}{C^2\hbar^2}$$

$$\alpha^2 = \frac{m\omega}{\hbar} \qquad (34)$$

Defining [see equation (16) and (34)

$$\lambda = C_3 = \frac{K^2}{\alpha^2} = \frac{(E^2 - m_o^2 c^4)}{c^2\hbar^2 m\omega}\hbar \qquad (35)$$

$$u = He^{-\frac{y^2}{2}}\bar{u} = u = u^/ = \left(H^- e^{-\frac{y^2}{2}} - yHe^{-\frac{y^2}{2}}\right)$$

$$u^" = \ddot{u} = \left(H^" - yH^/ - H - yH^/ + y^2 H\right)e^{-\frac{y^2}{2}} =$$

$$\left(H^" - 2yH^/ + (y^2 - 1)He^{-\frac{y^2}{2}}\right. \qquad (36)$$

A direct insertion of equation (36) and equation (3) and (35) in equation (32) yields

$$\ddot{H} - 2y\dot{H} + (y^2 - 1)H + \left(\dot{C_3} - y^2 - \frac{c^2}{y^2}\right)H = 0$$

$$\ddot{H} - 2y\dot{H} + (\lambda - 1)\dot{H} - \frac{c^2}{y^2}H = 0 \qquad (37)$$

$$H = \sum_s a_s y^s \;\;, \; \dot{H} = \sum_s s a_s y^{s-1} \;\;, \;\; \ddot{H} = \sum_s s(s-1)\; a_s y^{s-2} \qquad (38)$$

$$\sum_s s(s-1)\; a_s y^{s-2} - 2\sum_s a_s y^s + \sum_s(\lambda - 1)a_s y^s - c_2\; \sum_s a_s y^{s-2} = 0 \qquad (39)$$

To make all term raised to the power s, one can replace sbysts in the first and last terms on the left hand side of equation (39), to get

$$[\sum_s s(-1)(s+1)a_{s+2}y^s + (\lambda - 2s - 1)a_s y^s - \sum c_2\, a_{s+2}y^s = 0 \qquad (40)$$

Equating the coefficients of equal powers

$$[(s+2)(s+1) - c_2]a_{s+2} + (\lambda - 2s - 1)a_s = 0 \qquad (41)$$

Since the wave function $\psi$, u are finite. Thus the series must be finite, such that the last term is (s=n).

There fore

$$a_n \neq 0\,, \quad a_{n+1} = 0 \qquad a_{n+2} = 0 \qquad (42)$$

From equation (41)

$$a_{s+2} = \frac{(2s+1-\lambda)a_s}{(s+1)(s+2) - c_2} \qquad (43)$$

Setting (s=n)

$$0 = a_{n+2} = \frac{(2n+1-\lambda)a_n}{(n+1)(n+2) - c_2} \qquad (44)$$

Since $a_n \neq 0$ , it follows that

$$\lambda = 2n + 1 \qquad (45)$$

In view of equation (35)

$$\lambda = \frac{(E - (\frac{m_o^2 c^4}{E})}{\hbar\omega} \qquad (46)$$

Thus in view of equations (45, 46)

$$E - \frac{m_o^2\;\; c^4}{E} = (2n+1)\hbar\omega \qquad (47)$$

For very small rest mass or very large energy resulting from the nuclear potential is given by

$$E_n = E = (2N + 1)\hbar\omega \qquad (48)$$

This describes the effect of the nuclear electric field on nucleons energies. Another additional energy is effect of the agnatic field. This can be achieved by using perturbation theory. To do this, one can find the nuclear magnetic flux nuclear magnetic moment $\mu$ , which is define in term of the current i and area A to be:

$$\mu = iA = i(\pi r^2) = \frac{-e}{2m}\, \underline{L} \qquad (49)$$

Thus

$$i = \frac{\mu}{\pi r^2} \qquad (50)$$

But the magnetic flux density B can be found due to a circulation of current i in a circular path of the radius r, in a medium having magnetic permeability $\mu_o$ . Thus, B has given by:

$B = \frac{\mu_o \mu}{2\pi r^3} = \frac{\mu_o e}{4\pi m r^3} L$ (51)

The Interaction potential between nucleon spin, $\mu_{s,}$ and nuclear magnetic field is given by:

$V_m = -\mu_s . B = -\frac{e}{m} S.B$ (52)

$V_m = \frac{\mu_o e^2}{4\pi m r^3} S.L = \frac{\mu_o^2 e^2}{4\pi m r^3} L.S = V.(r) L.S$ (53)

There for the a additional magnetic moment gained by the nucleons is given by

$E_s = \int \bar{u}_i V_m u_j dr = \left(\int \bar{u}_i V_o(r) u_j d\right) L.S = V_o ij L.S = V_s L.S$ (54)

Where

$V_o == \frac{\mu_o^2 e^2}{4\pi m r^3}, \quad V_s = V_o ij$ (55)

But the total orbital angular momentum is given by

J=L+S (56)

There for

$\underline{j}.\underline{j} = (\underline{L} + \underline{S})(\underline{L} + \underline{S}) = \underline{L}.\underline{L} + \underline{S}.\underline{S} = 2\underline{L}.\underline{S} = L^2 + S^2 + 2L.S$

$\underline{L}.\underline{S} = \frac{1}{2}(j^2 - L^s - s^2) = \frac{\hbar^2}{2}(j(j+1) - l(l+1) - s(s+1)$

$= \frac{\hbar^2}{2}[j^2 + j - l^2 - l - s^2 - s)$ (56)

There fore

J=L+s (57)

$\underline{L}.\underline{S} = \frac{\hbar^2}{2}[l^2 + 2ls + s^2 + s - l^2 - l - s^2 - s) = \frac{\hbar^2}{2}[2ls]$

$\underline{L}.\underline{S} = \hbar^2 ls$ (58)

Hence

$L.S = \frac{\hbar^2}{2} L, \quad s = \frac{1}{2}, \quad j = l + \frac{1}{2}$

$.S = \frac{\hbar^2}{2} L, \quad s = -\frac{1}{2}, \quad j = l - \frac{1}{2}$ (59)

Thus from equation (48), (50) and (59), the total energy is given

$E = E_n + E_s = (2n+1)\hbar\omega \pm \frac{\hbar^2 V_s}{2} L$ (60)

The magic number can be explained using this expression as shown by M. GO pert Mayer and by D Haxel J.Jensen and H.suess.

L= n, n-2, n-4, … n-2i , i= 0,1,2,3

| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| L | 0 | 1 | 0, 2 | 1, 3 | 0, 2, 4 | 1, 3, 5 | 0, 2, 4,6 |

$n = 0 \qquad (n_x, \ n_y, n_z)$

$(0, 0,0) -= proton, Neutron$

$$n = 1 \qquad (n_x, \ n_y, n_z), (1, \ 0, 0), (0, \ 1, 0), (0, \ 0, 1) \ 3 \begin{bmatrix} (1,0,0) \ \dots\dots\dots\dots \frac{p}{n} \\ (0,1,0) \ \dots\dots\dots\dots \frac{p}{n} \\ (0,0,1) \ \dots\dots\dots\dots \frac{p}{n} \end{bmatrix} 6$$

$n = 2 \qquad (n = n_x + \ n_y, n_z)$

*[Rewais,* **8(4): April 2021]** **ISSN 2348 – 8034**
**DOI: https://doi.org/10.29121/gjesr.v8.i04.2021.1** **Impact Factor- 5.070**

$$6\begin{bmatrix}(1,1,0)\ldots\ldots\ldots\ldots\ldots\frac{p}{n}\\(1,0,1)\ldots\ldots\ldots\ldots\ldots\frac{p}{n}\\(0,1,1)\quad\ldots\ldots\ldots..\frac{p}{n}\\(2,0,0)\ldots\ldots\ldots\ldots\frac{p}{n}\\(0,2,0)\ldots\ldots\ldots\ldots\frac{p}{n}\\(0,0,2)\ldots\ldots\ldots\ldots..\frac{p}{n}\end{bmatrix}12$$

$n = 3 \qquad (n = n_x + \ n_y + n_z)$

$$10\begin{vmatrix}(1,1,1)\ldots\ldots\ldots\ldots\frac{p}{n}\\(2,1,0)\ldots\ldots\ldots.\frac{p}{n}\\(2,0,1)\ldots\ldots\ldots\frac{p}{n}\\(2,0,1)\ldots\ldots\ldots\frac{p}{n}\\(1,2,0)\ldots\ldots..\frac{p}{n}\\(0,2,1)\ldots\ldots..\frac{p}{n}\\(1,0,2)\ldots\ldots..\frac{p}{n}\\(3,0,0)\ldots\ldots..\frac{p}{n}\\(0,3,0)\ldots\ldots..\frac{p}{n}\\(0,0,3)\ldots\ldots..\frac{p}{n}\end{vmatrix}20$$

$j = L + \frac{1}{2}, \qquad L - \frac{1}{2}$

Splitting

$s(L = 0): j = \frac{1}{2} \qquad 2j + 1$

$p(L = 1) : j = 1 + \frac{1}{2} = \frac{3}{2} \qquad 2j + 1 = 4$

$j = 1 - \frac{1}{2} = \frac{1}{2} \qquad 2j + 1 = 2$

$d(l = 2) = j = 2 + \frac{1}{2} = \frac{5}{2} \qquad 2j + 1 = 6$

$j = 2 - \frac{1}{2} = \frac{3}{2} \qquad 2j + 1 = 4$

$F(L = 3) = j = 3 + \frac{1}{2} = \frac{7}{2} \qquad 2j + 1 = 8$

$j = 3 - \frac{1}{2} = \frac{5}{2} \qquad 2j + 1 = 6$

$g(L = 4) = j = 4 + \frac{1}{2} = \frac{9}{2} \qquad 2j + 1 = 10$

$j = 4 - \frac{1}{2} = \frac{7}{2} \qquad 2j + 1 = 8$

$h(j = 5): j = 5 + \frac{1}{2} = \frac{11}{2} \qquad 2j + 1 = 12$

$j = 5 - \frac{1}{2} = \frac{9}{2} \qquad 2j + 1 = 10$

## III. CONCLUSION

The proposed model shows the possibility of using potential dependent Klein-Gordon equation to find the magic number. This shows that, this equation can describe fermions as well as bosons.

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